

Transverse momentum broadening and gauge invariance

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Abstract. In the framework of the soft-collinear effective theory, we present a gauge invariant definition of the transverse momentum broadening probability of a highly-energetic collinear quark in a medium and consequently of the jet quenching parameter \hat{q} .

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INTRODUCTION

Jet quenching occurs when in a heavy-ion collision an energetic parton propagating in one light-cone direction loses sufficient energy that few high momentum hadrons are seen in the final state, where in the vacuum there would be a jet. In this context, a parton is considered highly energetic when its momentum Q is much larger than any other energy scale, including those characterizing the medium. Jet quenching, which has been observed at RHIC [1] and at LHC [2], manifests itself in many ways. In particular, the hard partons produced in the collision lose energy and change direction of their momenta. This last phenomenon goes under the name of transverse momentum broadening.

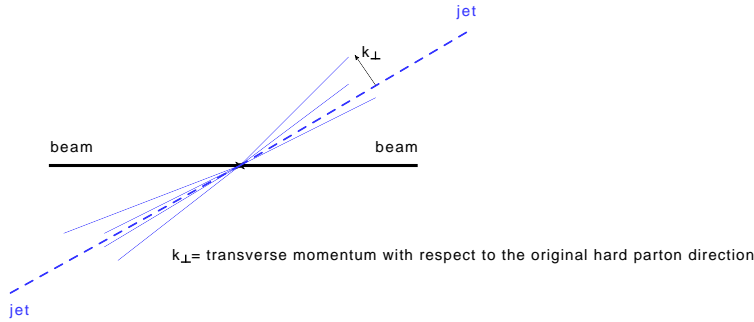


FIGURE 1. Kinematics of transverse momentum broadening.

A way to describe the transverse momentum broadening is by means of the probability, $P(k_\perp)$, that after propagating through the medium for a distance L ($\rightarrow \infty$) the hard parton acquires a transverse momentum k_\perp (see Fig. 1): $\int \frac{d^2 k_\perp}{(2\pi)^2} P(k_\perp) = 1$. A related quantity is the jet quenching parameter, \hat{q} , which is the mean square transverse momen-

tum picked up by the hard parton per unit distance traveled:

$$\hat{q} = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp). \quad (1)$$

In the following, we will review the derivation of a gauge invariant expression for $P(k_\perp)$ in the case of the propagation of a highly energetic quark. The original detailed derivation can be found in [3].

SCALES AND EFFECTIVE FIELD THEORY

We consider a highly energetic quark of momentum Q propagating along one light-cone direction $\bar{n} = (1, 0, 0, -1)/\sqrt{2}$. The light-cone momentum coordinates are $q^+ = \bar{n} \cdot q$, $q^- = n \cdot q$, with $n = (1, 0, 0, 1)/\sqrt{2}$, and q_\perp , which is the momentum component that is transverse with respect to the light-cone directions n and \bar{n} , see Fig. 2. If the quark propagates in a medium whose energy scales are much smaller than Q , then we can define a parameter $\lambda \ll 1$, which is the ratio of the energy scale characterizing the medium and Q . This small parameter may serve to classify the different modes of the propagating quark and interacting gluons.

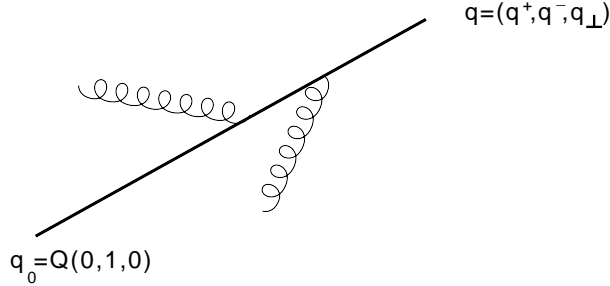


FIGURE 2. Parton momentum in light-cone coordinates.

We assume that the quark, after traveling along the medium, undergoes a transverse momentum broadening of order $Q\lambda$. If the virtuality of the quark is small, i.e. of order $Q^2\lambda^2$, then the parton has momentum $q \sim Q(\lambda^2, 1, \lambda)$ and is called collinear. We set up to describe the propagation of a single collinear quark in the medium. A collinear quark may scatter in the medium with ultrasoft gluons, whose momenta scale like $Q(\lambda^2, \lambda^2, \lambda^2)$, with Glauber gluons, whose momenta scale like $Q(\lambda^2, \lambda, \lambda)$ or $Q(\lambda^2, \lambda^2, \lambda)$ or with soft gluons scaling like $Q(\lambda, \lambda, \lambda)$ through the emission of virtual hard-collinear quarks scaling like $Q(\lambda, 1, \lambda)$. The relevant degrees of freedom are shown in Fig. 3.

The effective field theory that describes the propagation of a collinear quark in the \bar{n} light-cone direction is the soft-collinear effective theory (SCET) [4] coupled to Glauber gluons [5]. After rescaling the quark field by $\xi_{\bar{n}} \rightarrow e^{-iQx^+} \xi_{\bar{n}}$, the Lagrangian may be organized as an expansion in λ :

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \not{n} \cdot D \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \frac{D_\perp^2}{2Q} \not{n} \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i \frac{g F_\perp^{\mu\nu}}{4Q} \gamma_\mu \gamma_\nu \not{n} \xi_{\bar{n}} + \dots, \quad (2)$$

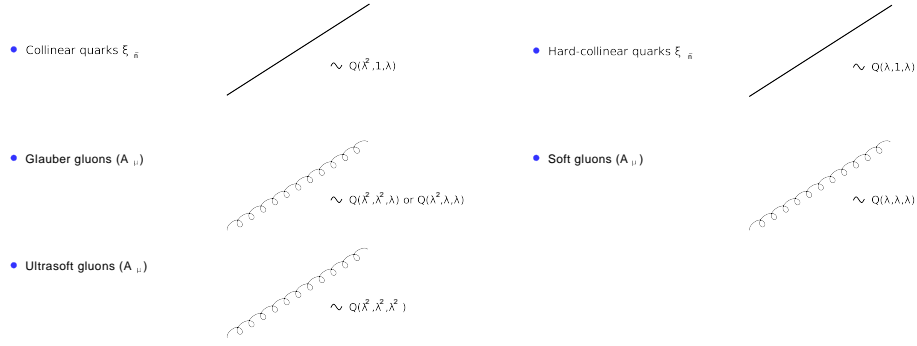


FIGURE 3. Relevant degrees of freedom.

where $iD_\mu = i\partial_\mu + gA_\mu$ and $F_\perp^{\mu\nu} = i[D_\perp^\mu, D_\perp^\nu]/g$ is the gluon field strength. The fragmentation of the collinear quark into collinear partons is not taken into account by the above Lagrangian; a preliminary study of this effect can be found in [6].

MOMENTUM BROADENING IN COVARIANT GAUGES

Collinear and hard-collinear quark fields, $\xi_{\bar{n}}(x)$, scale in the same way. The operators $\bar{n} \cdot \partial$ and ∇_\perp scale like $Q\lambda^2$ and $Q\lambda$ respectively when acting on a collinear field $\xi_{\bar{n}}(x)$, and both scale like $Q\lambda$ when acting on a hard-collinear field $\xi_{\bar{n}}(x)$. Soft gluon fields scale like $Q\lambda$ and ultrasoft gluon fields scale like $Q\lambda^2$, for they are homogeneous in the soft and ultrasoft scale respectively. In contrast, the power counting of Glauber gluons depends on the gauge. The equations of motion require $A^+(x)$ to scale like $Q\lambda^2$. In a covariant gauge, if the gluon field is coupled to a homogeneous soft source, this also implies that $A_\perp(x) \sim Q\lambda^2$. The leading order Lagrangian in λ is then

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \not{n} \bar{n} \cdot D \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \frac{\nabla_\perp^2}{2Q} \not{n} \xi_{\bar{n}}. \quad (3)$$

Because ultrasoft gluons decouple at lowest order from collinear quarks through the field redefinition $\xi_{\bar{n}}(x) \rightarrow \text{P exp} \left[ig \int_{-\infty}^{x^-} dy \bar{n} \cdot A_{\text{us}}(x^+, y, x_\perp) \right] \xi_{\bar{n}}(x)$, where P stands for the path ordering operator, only one relevant vertex involving either Glauber or soft gluons has to be taken into account:

$$\text{Diagram: A horizontal line (quark) connected to a vertical wavy line (gluon) labeled } A^+. \quad = igT^a \bar{n}^\mu \not{n}.$$

The transverse momentum broadening probability is then given by the imaginary part of the differential scattering amplitude

taken for $k_\perp \neq 0$ and normalized by the number of collinear quarks in the medium. The scattering amplitude has the form (evaluated on a background of gluon fields)

$$\int \prod_i \frac{d^4 q_i}{(2\pi)^4} \cdots \frac{iQ}{2Qq_2^+ - q_{2\perp}^2 + i\epsilon} \not{n} A^+(q_2 - q_1) \not{n} \frac{iQ}{2Qq_1^+ - q_{1\perp}^2 + i\epsilon} \not{n} A^+(q_1 - q_0) \not{n} \xi_{\bar{n}}(q_0),$$

where the Dirac spinor $\xi_{\bar{n}}(q_0)$ satisfies $\not{n} \xi_{\bar{n}}(q_0) = 0$ and is normalized as $\xi_{\bar{n}}^\dagger(q_0) \xi_{\bar{n}}(q_0) = \sqrt{2}Q$. For Glauber gluons, the free propagator may be approximated by (e.g. in Feynman gauge)

$$D_{\mu\nu}(k) = D(k^2)g_{\mu\nu} \approx D(k_\perp^2)g_{\mu\nu}, \quad (4)$$

which implies that the scattering amplitude in coordinate space is at leading order

$$\int dy^+ d^2 y_\perp \prod_i dy_i^- \cdots \theta(y_3^- - y_2^-) A^+(y^+, y_2^-, y_\perp) \theta(y_2^- - y_1^-) A^+(y^+, y_1^-, y_\perp) \xi_{\bar{n}}(q_0). \quad (5)$$

The same result also holds when considering the case of (hard-)collinear quarks interacting with soft gluons.

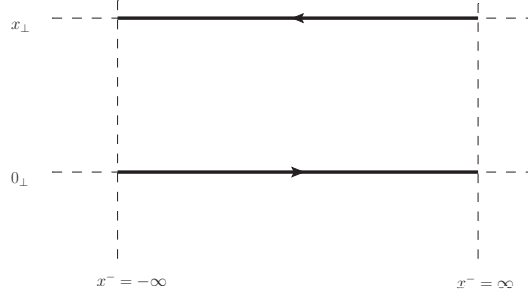


FIGURE 4. Wilson lines contributing to $P(k_\perp)$ in a covariant gauge.

Because (5) is just a term in the expansion of the Wilson line $W[y^+, y_\perp] = \text{P exp} \left[ig \int_{-L/\sqrt{2}}^{L/\sqrt{2}} dy^- A^+(y^+, y^-, y_\perp) \right]$ for $L \rightarrow \infty$, the transverse momentum broadening probability of a quark in covariant gauges is given by

$$P(k_\perp) = \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \text{Tr} \left\{ W^\dagger[0, x_\perp] W[0, 0] \right\} \right\rangle, \quad (6)$$

where $\langle \dots \rangle$ denotes a field average. The Wilson lines of (6) are shown in Fig. 4. The relation between jet quenching and Wilson lines oriented along one of the light-cone directions was derived within different approaches in [7] and within SCET in [8]. Clearly the above expression is, in general, not gauge invariant (e.g. in the light-cone gauge $A^+ = 0$, one would have $W = 1$).

MOMENTUM BROADENING IN LIGHT-CONE GAUGE

In the light-cone gauge $A^+ = 0$, the free gluon propagator reads

$$D_{\mu\nu}(k) = D(k^2) \left(g_{\mu\nu} - \frac{k_\mu \bar{n}_\nu + k_\nu \bar{n}_\mu}{[k^+]} \right). \quad (7)$$

For Glauber gluons $k_\perp/[k^+] \sim 1/\lambda$, which leads to an enhancement of order λ in the singular part of the propagator. Moreover, because of the $k_\perp/[k^+]$ singularity, one can write [9] $A_\perp(x^+, x^-, x_\perp) = A_\perp^{\text{cov}}(x^+, x^-, x_\perp) + A_\perp^{\text{sin}}(x^+, x^-, x_\perp)$, where $A_\perp^{\text{cov}}(x)$ contributes to the non-singular part of the propagator and vanishes at $x^- = \pm\infty$, while $A_\perp^{\text{sin}}(x^+, x^-, x_\perp) = \theta(x^-)A_\perp(x^+, \infty, x_\perp) + \theta(-x^-)A_\perp(x^+, -\infty, x_\perp)$ with $A_\perp(x^+, \pm\infty, x_\perp) = -\nabla_\perp \phi^\pm(x^+, x_\perp)$. The field A_\perp does not vanish at infinity where it becomes pure gauge, for the field tensor does (the energy of the gauge field is finite).

In the $A^+ = 0$ gauge, the scaling of the Glauber fields appearing in the Lagrangian changes to $A_\perp^{\text{cov}} \sim Q\lambda^2$ and $A_\perp^{\text{sin}} \sim Q\lambda$. The leading order Lagrangian in λ is then

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \not{n} \bar{n} \cdot \partial \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} \frac{(\nabla_\perp + ig A_\perp^{\text{sin}})^2}{2Q} \not{n} \xi_{\bar{n}}, \quad (8)$$

where gluons are just Glauber gluons. The relevant vertices are now two

$$\begin{aligned} & \text{Diagram 1: A horizontal line with momentum } q \text{ on the left and } q' \text{ on the right. A wavy line labeled } A_\perp \text{ connects them.} \\ & = -ig \frac{q'_\perp \cdot A_\perp^{\text{sin}}(q' - q) + A_\perp^{\text{sin}}(q' - q) \cdot q_\perp}{2Q} \not{n}, \\ \\ & \text{Diagram 2: A horizontal line with momentum } q \text{ on the left and } q'' \text{ on the right. Two wavy lines labeled } A_\perp \text{ connect them.} \\ & = -\frac{ig^2}{2Q} \int \frac{d^4 q'}{(2\pi)^4} A_\perp^{\text{sin}}(q'' - q') A_\perp^{\text{sin}}(q' - q) \not{n}. \end{aligned}$$

From the vertices one constructs the scattering amplitude (on the left of the cut)

$$\begin{aligned} & \text{Diagram: A horizontal line with momentum } q_0 = Q(0, 1, 0) \text{ on the left and } (k_\perp^2/2Q, k^-, k_\perp) \text{ on the right. It is connected to a series of wavy lines labeled } A_\perp. \\ & \text{The wavy lines are numbered 1, 2, 3, 4, 5, ..., n-1, n.} \\ & = G_n(k^-, k_\perp). \end{aligned}$$

The function G_n is a convolution of G_{n-j}^+ , which involves only fields at $x^- = \infty$ and G_j^- , which involves only fields at $x^- = -\infty$:

$$G_n(k^-, k_\perp) = \sum_{j=0}^n \int \frac{d^4 q}{(2\pi)^4} G_{n-j}^+(k^-, k_\perp, q) \frac{iQ \not{n}}{2Qq^+ - q_\perp^2 + i\varepsilon} G_j^-(q). \quad (9)$$

The computation is done by solving recursively the equation (analogously for $G_n^+(q)$)

$$G_n^-(q) = \int \frac{d^4 q'}{(2\pi)^4} G_{n-1}^-(q') \overbrace{\text{---}}^{q'} \text{---}^q + \int \frac{d^4 q''}{(2\pi)^4} G_{n-2}^-(q'') \overbrace{\text{---}}^{q''} \text{---}^q, \quad (10)$$

writing the differential amplitude as

$$\frac{1}{L^3 \sqrt{2Q}} \int \frac{dk^+}{2\pi} \int \frac{dk^-}{2\pi} 2\pi Q \delta(2Qk^+ - k_\perp^2) \bar{\xi}_{\bar{n}}(q_0) G_m^\dagger(k^-, k_\perp) \not{n} G_n(k^-, k_\perp) \xi_{\bar{n}}(q_0),$$

and eventually summing over all m and n . The expression of the transverse momentum broadening probability in light-cone gauge then reads

$$P(k_\perp) = \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \text{Tr} \left\{ T^\dagger[0, -\infty, x_\perp] T[0, \infty, x_\perp] T^\dagger[0, \infty, 0] T[0, -\infty, 0] \right\} \right\rangle, \quad (11)$$

where $T[0, \pm\infty, x_\perp] = \text{P exp} \left[-ig \int_{-\infty}^0 ds l_\perp \cdot A_\perp(0, \pm\infty, x_\perp + sl_\perp) \right]$ (for the definition of T see also [10]). The transverse vector l_\perp is arbitrary. The Wilson lines of (11) are shown in Fig. 5.

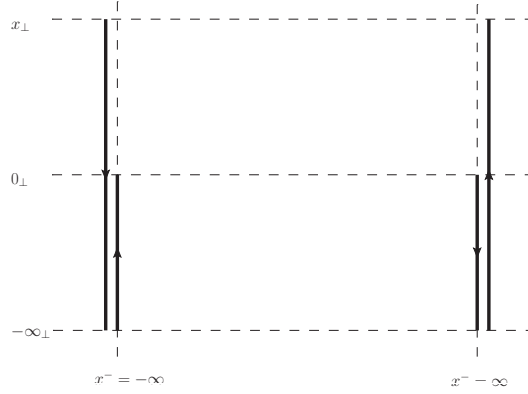


FIGURE 5. Wilson lines contributing to $P(k_\perp)$ in light-cone gauge. We have chosen $l_\perp \parallel x_\perp$.

GAUGE INVARIANT MOMENTUM BROADENING

Combining the results in covariant and light-cone gauge for $L \rightarrow \infty$, we obtain a gauge invariant expression for $P(k_\perp)$, which reads

$$P(k_\perp) = \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \text{Tr} \left\{ T^\dagger[0, -\infty, x_\perp] W^\dagger[0, x_\perp] T[0, \infty, x_\perp] \right. \right. \\ \left. \left. \times T^\dagger[0, \infty, 0] W[0, 0] T[0, -\infty, 0] \right\} \right\rangle. \quad (12)$$

The Wilson lines of (12) are shown in Fig. 6. Note that the fields are path ordered but not time ordered as in usual Wilson loops [11]. This difference should not be surprising

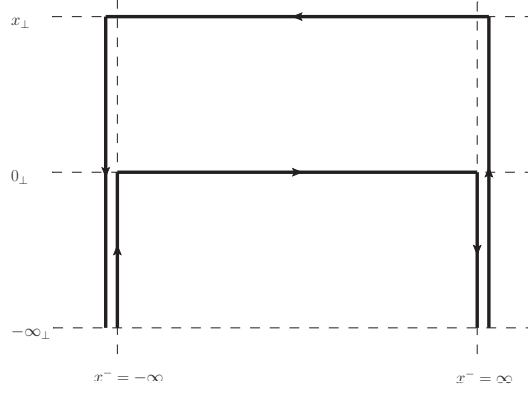


FIGURE 6. Wilson lines contributing to the gauge invariant expression of $P(k_\perp)$ given in (12). We have chosen $l_\perp \parallel x_\perp$. The fields at $x^- = \infty$ are contiguous while those at $x^- = -\infty$ are not.

since it reflects the fact that $P(k_\perp)$ describes the propagation of a single particle, while usual Wilson loops describe the propagation of a particle-antiparticle pair.

The expression of $P(k_\perp)$ may be simplified into

$$P(k_\perp) = \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \text{Tr} \left\{ [0, x_\perp]_- W^\dagger[0, x_\perp] [x_\perp, 0]_+ W[0, 0] \right\} \right\rangle, \quad (13)$$

where $[x_\perp, y_\perp]_\pm = \text{P exp} \left[-ig \int_1^0 ds (y_\perp - x_\perp) \cdot A_\perp(0, \pm\infty, x_\perp + s(y_\perp - x_\perp)) \right]$, because contiguous adjoint lines cancel, fields separated by space-like intervals commute and because of the cyclicity of the trace. The Wilson lines of (13) are shown in Fig. 7.

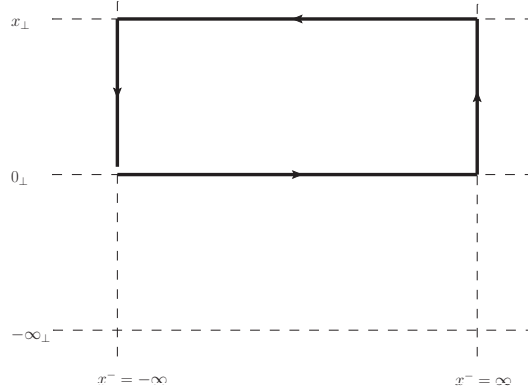


FIGURE 7. Wilson lines contributing to the gauge invariant expression of $P(k_\perp)$ given in (13). The fields in $(0, -\infty, 0)$ are not contiguous.

The obtained expression for $P(k_\perp)$ does not depend on l_\perp . It is also gauge invariant. In fact, under a gauge transformation Ω , $\text{Tr} \{ T^\dagger[0, -\infty, x_\perp] W^\dagger[0, x_\perp] \cdots T[0, -\infty, 0] \}$ transforms to $\text{Tr} \{ \Omega[0, -\infty, -\infty l_\perp] T^\dagger[0, -\infty, x_\perp] W^\dagger[0, x_\perp] \cdots T[0, -\infty, 0] \Omega^\dagger[0, -\infty, -\infty l_\perp] \}$, which is equal to the original expression, $\text{Tr} \{ T^\dagger[0, -\infty, x_\perp] W^\dagger[0^+, x_\perp] \cdots T[0, -\infty, 0] \}$, after noticing that the fields in $\Omega[0, -\infty, -\infty l_\perp]$ commute with all the others (because of space-like separations) and after using the cyclicity of the trace.

CONCLUSION

Having derived the transverse momentum broadening probability, $P(k_\perp)$, we are in the position to write the jet quenching parameter \hat{q} in a manifestly gauge invariant fashion:

$$\hat{q} = \int \frac{d^2 k_\perp}{(2\pi)^2} d^2 x_\perp dx^- e^{ik_\perp \cdot x_\perp} \frac{\sqrt{2}}{N_c} \left\langle \text{Tr} \left\{ [0, x_\perp]_- U_{x_\perp}^\dagger[x^-, -\infty] gF_\perp^{+i}(0, x^-, x_\perp) \right. \right. \\ \left. \left. \times U_{x_\perp}^\dagger[\infty, x^-][x_\perp, 0]_+ U_{0_\perp}[\infty, 0] gF_\perp^{+i}(0, 0, 0) U_{0_\perp}[0, -\infty] \right\} \right\rangle, \quad (14)$$

where the fields $F_\perp^{+i} = \bar{n} \cdot \partial A_\perp^i - \nabla_\perp^i A^+ + ig[A^+, A_\perp^i]$ come from the derivatives, ∇_\perp^i , acting on the Wilson lines, and $U_{x_\perp}[x^-, y^-] = \text{P exp} \left[ig \int_{y^-}^{x^-} dz^- A^+(0, z^-, x_\perp) \right]$. We recall that the above expression holds when the integral over k_\perp has an ultraviolet cut-off of order $Q\lambda$, which is the size of the transverse momentum broadening that we have been considering.

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